BUSINESS SCHOOL

SKEMA

Corporate Finance
PGE M1 Fall 2026

Jason Kang

Session 8: CAPM

Housekeeping

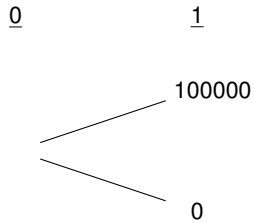


About me

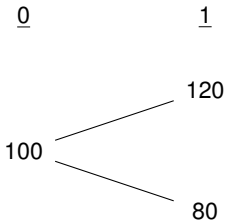
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Plan For Today

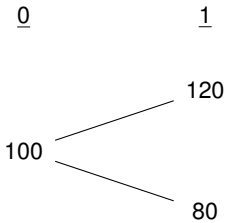
- Derivation (Supplementary)
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- Appendix: CAPM in Practice
- Appendix: Re-levering β



Suppose you have a lottery with payoffs as described above. The probability of the upper node is $p = 0.001$ such that the expected payoff of this gamble is equal to 100. Would you be willing to pay 100 for this bet?



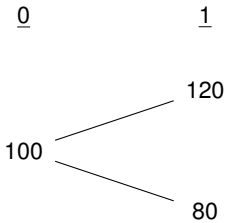
Suppose a stock that will either take values 120 or 80 in one year has a market price of 100 today. Assume $r^f = 0\%$, and the true probability of the good state is equal to $p = 0.6$.



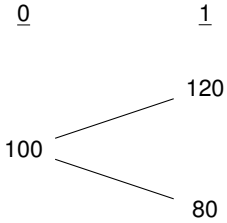
The expected value of the stock's payoff is 104, but we observe a market price of 100. One way to justify the market price of 100 despite the true "physical" probability $p = 0.6$ is if investors had distorted probability assessments, $q = 0.5$,

$$0.5(120) + 0.5(80) = 100$$

We call q the risk-neutral probabilities.



This implies that to reconcile what we observe in the market with the true probability of events, there is an adjustment that accounts for investors' attitudes towards risk.



$$= 0.6 \times \overbrace{\left(\frac{0.5}{0.6}\right)}^{\approx 0.83} \times 120 + 0.4 \times \overbrace{\left(\frac{0.5}{0.4}\right)}^{=1.25} \times 80$$

$$= 100$$

Asset Pricing Equation



Due to risk-aversion, investors put more weight on bad states of the world. Letting m denote the discount factor or the state-price density, the price of the asset P_t is given by

$$P_t = \sum_i \text{pr}_i m_{i,t+1} X_{i,t+1}$$

or in expectations,

$$P_t = \mathbb{E}_t \Big[m_{t+1} X_{t+1} \Big]$$

Asset Pricing Equation



Towards CAPM

Starting from the equation

$$P_t = \mathbb{E}_t\Big[m_{t+1}X_{t+1}\Big]$$

Divide both sides by P_t to express in terms of returns,

$$1 = \mathbb{E}_t\Big[m_{t+1}R_{t+1}\Big]$$

where R_{t+1} is the gross return on the asset.
We have, by $E(mR) = E(m)E(R) + cov(m, R)$, for any asset i

$$\mathbb{E}_t[R^i_{t+1}] - R^f = -R^f \mathrm{cov}(m_{t+1}, R^i_{t+1})$$

using $E(m) = 1/R^f$.

Towards CAPM

Recall, $m \approx 0.83$ in the good state and $m = 1.25$ in the bad state. An asset that gives a high payoff in the good state and a low payoff in the bad state (most assets except insurance) has a negative covariance with the discount factor. This means

$$\mathbb{E}_t[R^i_{t+1}] - R^f = -R^f \mathrm{cov}(m_{t+1}, R^i_{t+1})$$

is a positive number, and investors are compensated with high expected returns for holding risky assets that do poorly in bad states of the world that investors value the most.

Towards CAPM



Towards CAPM

We want to model m such that it is high in bad states and low in good states. What defines a "bad state" for a typical investor? A market downturn. So let m depend on the market return R^m:

$$m = a - bR^m, \qquad b > 0$$

Check in our tree. Take the stock as the "market" ($R^m = 1.2$ or 0.8, with $R^f = 1$). Matching $m = 0.83$ and $m = 1.25$ gives

$$b = \frac{1.25 - 0.83}{1.2 - 0.8} \approx 1.04, \qquad a \approx 2.08$$

The CAPM is an asset pricing model in which only market risk matters for m.

Towards CAPM

Assume a linear pricing model, $m = a - bR^m$ where R^m is the return on the market portfolio. Then for any asset i,

$$\begin{aligned}\mathbb{E}_t[R^i_{t+1}] - R^f &= -R^f \text{cov}(m_{t+1}, R^i_{t+1}) \\ &= R^f b \text{cov}(R^m_{t+1}, R^i_{t+1})\end{aligned}$$

Applying the equation to the market portfolio itself to pin down b,

$$\begin{aligned}\mathbb{E}_t[R^m_{t+1}] - R^f &= R^f b \text{cov}(R^m_{t+1}, R^m_{t+1}) \\ &= R^f b \text{var}(R^m_{t+1}) \\ \left(\frac{\mathbb{E}_t[R^m_{t+1}] - R^f}{\text{var}(R^m_{t+1})}\right) &= R^f b\end{aligned}$$

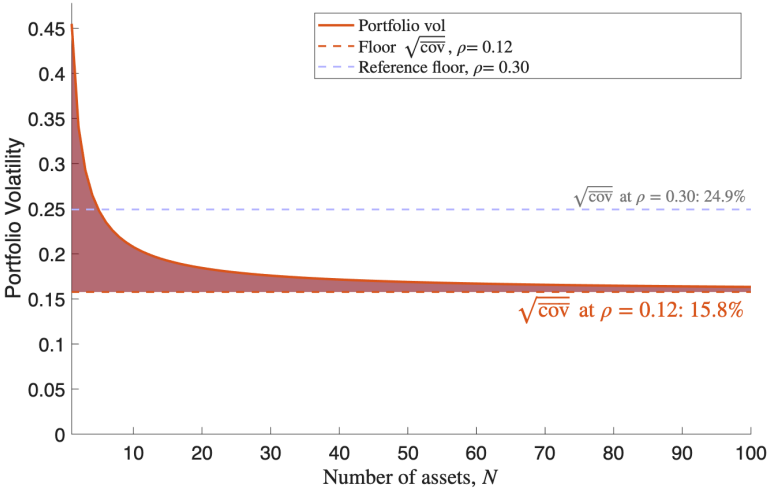
Plugging back into the top equation and re-arranging,

$$\mathbb{E}_t[R^i_{t+1}] - R^f = \frac{\text{cov}(R^m_{t+1}, R^i_{t+1})}{\text{var}(R^m_{t+1})}(\mathbb{E}_t[R^m_{t+1}] - R^f) = \beta^i(\mathbb{E}_t[R^m_{t+1}] - R^f)$$

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Portfolio Volatility: $\bar{\sigma} = 0.45,\ \bar{\rho} = 0.12$



Market Risk vs Idiosyncratic Risk



There are many names, but firm-specific, idiosyncratic, or diversifiable risk refers to good or bad news about the company itself.

Example. The risk of a CEO targeted by a foreign intelligence officer in an extramarital affair.

An investor, by holding a diversified portfolio of many stocks, can reduce his/her portfolio volatility to $\sqrt{\overline{\text{cov}}}$. We call the risk that investors cannot remove as market, systematic, or non-diversifiable risk.

Namely, these are events/news that contribute to the covariance between two assets.

Which type of risk is this?

- Covid-19
- A global recession / financial crises
- Failed clinical trial
- Product failure or recall
- Lawsuits
- Interest rate hikes
- Earthquakes
- New product launch
- CEO scandal

Compensation for Taking Risk



If an investor has a diversified portfolio of many stocks, adding one more stock only contributes to the average covariance of the portfolio, as its individual volatility largely washes out (diversifiable risk).

If the stock does well when the rest of your portfolio does well, this stock amplifies the risk of your portfolio.

If the stock does well when the rest of your portfolio does poorly, this stock reduces the risk of your portfolio by acting as insurance.

Compensation for Taking Risk



If the stock's own volatility does not contribute much towards the risk of the diversified portfolio, this means that we do not care about the total risk (volatility) of an individual stock, but only its covariance to our portfolio.

From the Appendix slides in Session 7, the two-fund separation theorem says investors should hold the market portfolio as it gives the best ratio of excess returns per unit of risk.
$\Leftrightarrow$ We should care about the stock's covariance to the market portfolio.

Measure of Covariance

The beta, or β, of an asset measures its sensitivity to market risk.

For stock *i*, its beta is

$$\beta^i = \frac{\text{cov}(R^i, R^m)}{\text{var}(R^m)} = \rho^{i,m} \times \frac{\sigma^i}{\sigma^m}$$

where

- R^i is the return on asset *i*
- R^m is the return on the market portfolio
- $\rho^{i,m}$ is the correlation between asset *i* and the market
- σ^i, σ^m are the volatilities of asset *i* and the market

Measure of Covariance

The beta, or β, of an asset measures its sensitivity to market risk.

For stock *i*, its beta is

$$\beta^i = \frac{\text{cov}(R^i, R^m)}{\text{var}(R^m)} = \rho^{i,m} \times \frac{\sigma^i}{\sigma^m}$$

Example. Suppose the correlation between KO and the market is equal to 0.33. KO has an annual volatility of 15%, and the market portfolio has an annual volatility of 18%. What is the beta of KO?

Measure of Covariance



Interpretation.

If the market portfolio goes up by 1 percentage point, we expect the stock to go up by β percentage points.

Example. Suppose the S&P 500 went up by 3%. By what percent do we expect the stock price of KO to go up by?

Beta's of Individual Companies

Based on 5-year daily data, 2021-2025.

Ticker	β	Ticker	β
JNJ	0.18	MSFT	1.09
CL	0.19	AAPL	1.17
PG	0.27	NFLX	1.26
KO	0.28	AMZN	1.43
MCD	0.38	UBER	1.44
SBUX	0.96	TSLA	2.03
DIS	1.00	NVDA	2.08
NKE	1.06		

The Capital Asset Pricing Model (CAPM)



The CAPM states that an individual company i's expected return can be completely explained by its covariance to the market portfolio. Expressing in net returns,

$$\mathbb{E}[r^i] = r^f + \beta^i \times \underbrace{(\mathbb{E}[r^m] - r^f)}_{\text{risk premium}}$$

We call $\mathbb{E}[r^m] - r^f$ the market risk premium. It is the compensation for bearing risk, the expected return in excess of holding the risk-free asset.

Example. What is $\mathbb{E}[r^i]$ if

- $\beta^i = 0$
- $\beta^i = 1$

Interpreting CAPM



An alternative interpretation of the CAPM is the required rate of return, or a company's equity cost of capital.

* As we have stressed repeatedly, investors only care about covariance to non-diversifiable risk, and not the total risk of individual stocks. The CAPM states that companies with identical market risk share the same equity cost of capital (what we use to discount cash flows by).

Formulas (so far)

The CAPM equation

$$\mathbb{E}[r^i] = r^f + \beta^i \times (\mathbb{E}[r^m] - r^f)$$

For stock i, its beta is

$$\beta^i = \frac{\mathrm{cov}(r^i, r^m)}{\mathrm{var}(r^m)} = \rho^{i,m} \times \frac{\sigma^i}{\sigma^m}$$

Example



Suppose the risk-free rate is 4.5%, and the expected return on the market is 12.5%.

a. Suppose the beta of MCD is 0.38. What is the expected return on MCD stock?
   7.54%

Example

Suppose the risk-free rate is 4.5%, and the expected return on the market is 12.5%.

b. Suppose the correlation between NKE and the market portfolio 0.54 and the volatility of NKE is 34.9%. If the volatility on the market portfolio is 17.7%, compute
   i The beta of NKE 1.0647
   ii The expected return on NKE 13.02%

Example



Suppose the risk-free rate is 4.5%, and the expected return on the market is 12.5%.

c. Assume the volatility on the market portfolio is 17.7% as before. If the beta of NFLX is 1.26, and the correlation between NFLX and the market portfolio is 0.51, compute the annual volatility of NFLX stock.
   43.73%

Example

Suppose the risk-free rate is 4.5%, and the market risk premium is 5%.

d. What is the required rate of return on investing in Disney. which has a $\beta_{\text{DIS}} = 1.0$?
9.5%

Example



Suppose the risk-free rate is 4.5%, and the expected return on the market is 12.5%.

e. What is the required rate of return on investing in gold. which has a $\beta = -0.5$?
0.5%

The Security Market Line (SML)

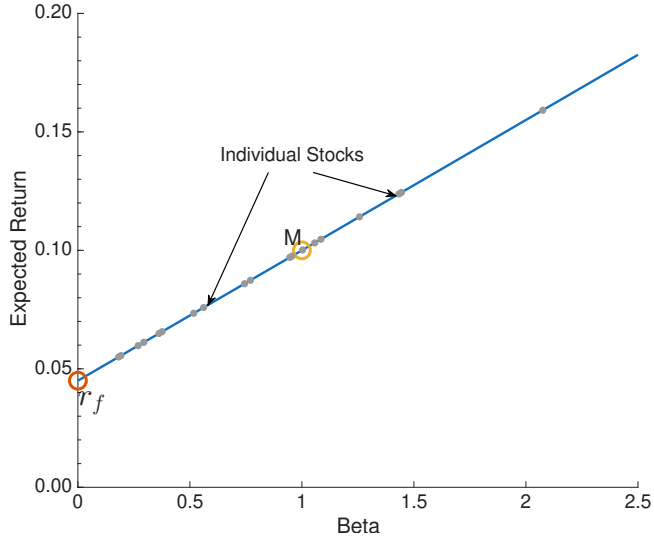


The CAPM is a strong statement. It says that the expected returns of an individual stock is completely captured by its market beta.
$\rightarrow$If AMZN has higher stock returns than KO, then it is because AMZN has higher market beta.

Graphically, the security market line (SML) plots the expected returns of individual stocks against their beta's.

According to the CAPM, the expected returns of all stocks should lie along a single line.

The Security Market Line



The Security Market Line

The formula for the SML is simply the CAPM,

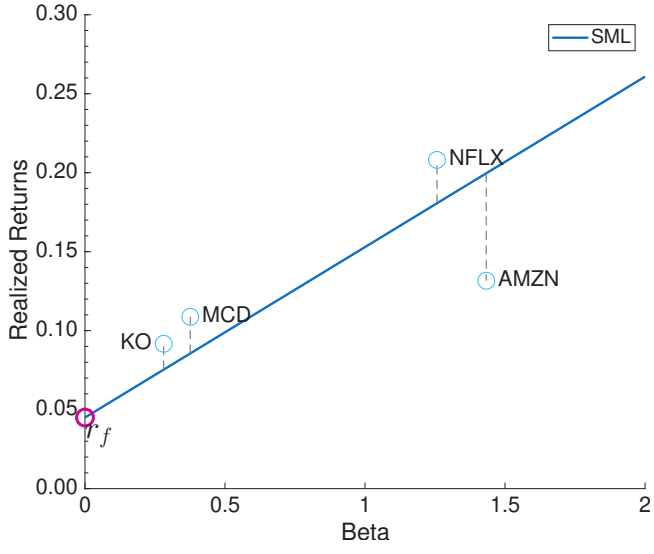
$$\mathbb{E}[r^i] = r^f + \beta^i \times (\mathbb{E}[r^m] - r^f)$$

What is the slope of the security market line?

What is the y-intercept?

How can you infer the market risk premium based on the graph of the SML?

The Security Market Line: Realized vs Expected



Measuring Performance



The CAPM is a model of expected returns but the previous plot showed that realized returns do not lie exactly on the SML. Does this mean the CAPM is wrong?

Define alpha as a measure of performance in excess of what is predicted/required by the CAPM,

$$\alpha_i \doteq r_i - \underbrace{\left[r^f + \beta_i \times (\mathbb{E}[r_m] - r^f)\right]}_{\text{CAPM}}$$

$\alpha > 0$ implies ?

Measuring Performance

Most hedge funds and mutual fund managers talk about positive alpha as a sign of "beating the market" because you are delivering returns higher than the fair compensation required for the level of risk you are taking.

Example. Suppose our portfolio has a beta of 1.5. The risk-free rate is 4.5% and the market risk premium is 5.5%. If our portfolio returns were 14% over the past year, what was the alpha of our portfolio, and how was our performance relative to what is predicted by the CAPM?

Measuring Performance



Again, the CAPM is a model of expected returns, so some stocks and portfolios earning positive/negative alpha is not a violation of CAPM. Remember, individual stocks carry idiosyncratic volatility (firm-specific news) that largely gets washed away in a diversified portfolio.

It becomes a problem if alpha's across all assets are not jointly zero, as it means the CAPM is systematically failing to capture an aspect of expected returns.

Example

- Suppose low beta stocks persistently have positive alpha year after year.

Exercise 1

Use the data in the following table of past annual returns to estimate:

	2004	2005	2006	2007	2008	2009
Market index	4%	−5%	7%	5%	0%	10%
Stock A	21%	7%	30%	−3%	−8%	25%

i the average return and volatility for stock A and the market

ii the covariance between stock A and the market

iii the beta of stock A.

Average Return and Volatility

Average return: $\bar{r} = \frac{1}{T}\sum r_t$. Volatility: $\sigma = \sqrt{\frac{1}{T-1}\sum(r_t - \bar{r})^2}$.

Year	Market	A	Mkt$-\bar{r}^M$	A$-\bar{r}^A$	$(\text{Mkt} - \bar{r}^M)^2$	$(A - \bar{r}^A)^2$
2004	0.04	0.21	0.005	0.09	0.000025	0.0081
2005	-0.05	0.07	-0.085	-0.05	0.007225	0.0025
2006	0.07	0.30	0.035	0.18	0.001225	0.0324
2007	0.05	-0.03	0.015	-0.15	0.000225	0.0225
2008	0.00	-0.08	-0.035	-0.20	0.001225	0.0400
2009	0.10	0.25	0.065	0.13	0.004225	0.0169
Sum	0.21	0.72			0.014150	0.1224
				Variance = Sum/5	0.00283	0.02448
				Volatility = $\sqrt{\text{Variance}}$	5.32%	15.65%

Average return: Market $= 0.21/6 = 3.5\%$, Stock A $= 0.72/6 = 12\%$

Sample Covariance



Using $\bar{r}^M = 3.5\%$ and $\bar{r}^A = 12\%$:

Year	Market	A	Mkt$-\bar{r}^M$	A$-\bar{r}^A$	Product
2004	0.04	0.21	0.005	0.09	0.00045
2005	-0.05	0.07	-0.085	-0.05	0.00425
2006	0.07	0.30	0.035	0.18	0.00630
2007	0.05	-0.03	0.015	-0.15	-0.00225
2008	0.00	-0.08	-0.035	-0.20	0.00700
2009	0.10	0.25	0.065	0.13	0.00845
				Sum of products	0.02420
				Covariance $= 0.0242/5$	0.00484

Beta of Stock A

$$\beta^A = \frac{\mathrm{cov}(r^A, r^M)}{\mathrm{var}(r^M)} = \frac{0.00484}{0.00283} \approx 1.71$$

Equivalently, $\beta^A = \rho^{A,M} \frac{\sigma^A}{\sigma^M}$ with $\rho^{A,M} = \frac{0.00484}{0.0532 \times 0.1565} \approx 0.58$, so $\beta^A = 0.58 \times \frac{15.65}{5.32} \approx 1.71$.

Interpretation: when the market moves by 1%, stock A moves by about 1.71% on average.

Exercise 2



Suppose the risk-free return is 4% and the market portfolio has an expected return of 10% and a volatility of 16%. Johnson and Johnson Corporation (Ticker: JNJ) stock has a 20% volatility and a correlation with the market of 0.06.

i What is J&J's beta with respect to the market?

ii Under the CAPM assumptions, what is its expected return?

Beta of JNJ

$$\beta^{JNJ} = \frac{\text{cov}(r^{JNJ}, r^{M})}{\text{var}(r^{M})} = \rho^{JNJ,M} \frac{\sigma^{JNJ}}{\sigma^{M}}$$

$$\beta^{JNJ} = 0.06 \times \frac{0.20}{0.16} = 0.075$$

Interpretation: JNJ is almost uncorrelated with the market, so it has very low systematic risk.

CAPM Expected Return of JNJ

$$\mathbb{E}[r^{JNJ}] = r^f + \beta^{JNJ}\left(\mathbb{E}[r^M] - r^f\right)$$

$$\mathbb{E}[r^{JNJ}] = 4\% + 0.075 \times (10\% - 4\%) = 4\% + 0.45\% = 4.45\%$$

Despite its 20% volatility, JNJ earns only slightly more than the risk-free rate, because most of its risk is diversifiable and not rewarded under the CAPM.

Exercise 3

Suppose Intel stock has a beta of 2.16, whereas Boeing stock has a beta of 0.69.

If the risk-free interest rate is 4% and the expected return of the market portfolio is 10%, what is the expected return of a portfolio that consists of 60% Intel stock and 40% Boeing stock, according to the CAPM?

$$\mathbb{E}[r^{INTC}] = 4\% + 2.16 \times (10\% - 4\%) = 16.96\%$$
$$\mathbb{E}[r^{BA}] = 4\% + 0.69 \times (10\% - 4\%) = 8.14\%$$

Expected Return of the Portfolio

$$\mathbb{E}[r^P] = 0.6 \times \mathbb{E}[r^{INTC}] + 0.4 \times \mathbb{E}[r^{BA}] = 0.6 \times 16.96\% + 0.4 \times 8.14\% = 13.432\%$$

Check using the portfolio beta:

$$\beta^P = 0.6 \times 2.16 + 0.4 \times 0.69 = 1.572$$

$$\mathbb{E}[r^P] = 4\% + 1.572 \times (10\% - 4\%) = 13.432\%$$

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How is β estimated in practice?

For stock *i*, its beta is

$$\beta^i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)} = \rho^{i,m} \times \frac{\sigma_i}{\sigma_m}$$

where r_i is the return on the stock and r_m is the return on the market portfolio.

In practice, we estimate it via an ordinary least-squares regression:

i. Collect past returns on the stock and the market
ii. Plot each period's stock return against the market return
iii. Fit the straight line that best describes the series of points

▷ The slope of that line is the estimated β

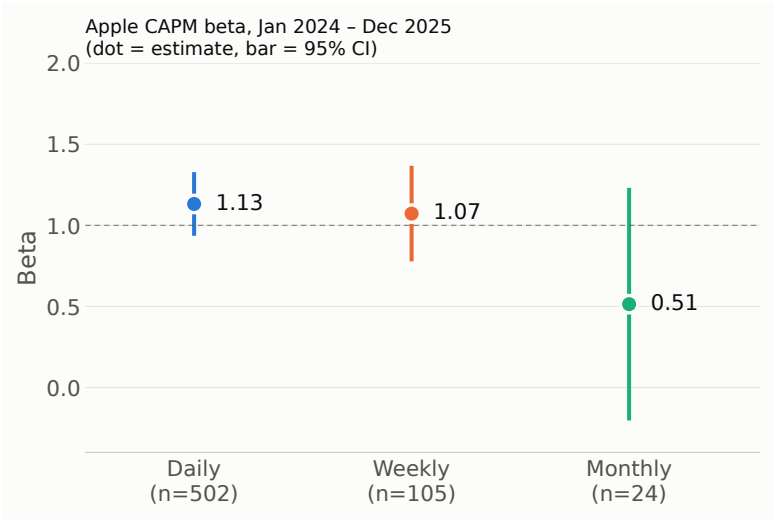

Monthly returns, Jan 2021 – Dec 2025 (60 months)
Slope = β = 1.05
Apple excess return (%)
Market excess return (%)
20
15
10
5
0
−5
−10
−10
−5
0
5
10

Apple CAPM beta by estimation window and frequency

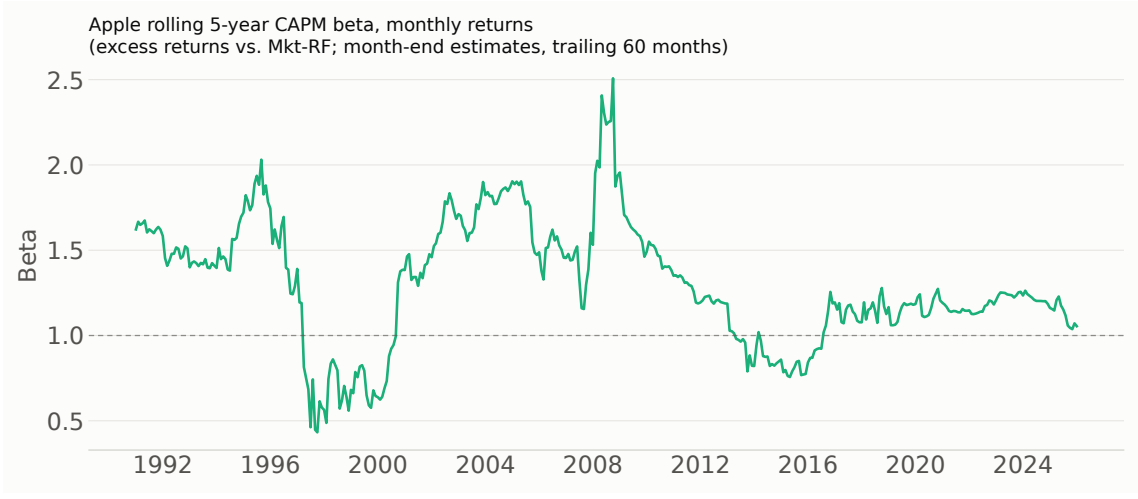
	Daily	Weekly	Monthly
Full sample (1986–2025)	1.21	1.13	1.32
Last 5 years	1.17	1.11	1.05
Last 2 years	1.13	1.07	0.51
Last 1 year	1.25	1.21	–

*Bloomberg terminal uses 2-year weekly with its own method of “correction”. Yahoo Finance uses 5-year monthly.

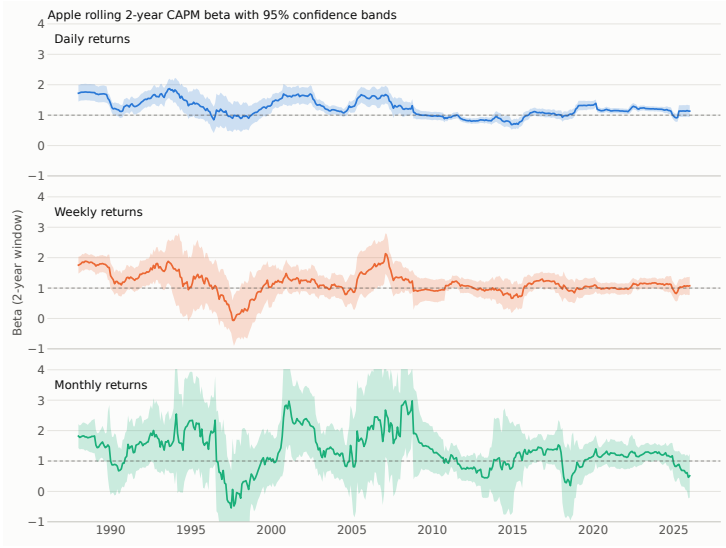
AAPL beta: 2024-2025



AAPL beta over rolling windows



AAPL beta over rolling windows



CAPM in practice

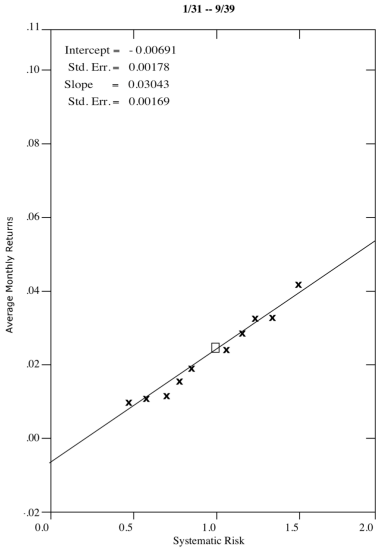


Figure 8 Average Excess monthly returns versus systematic risk for the 105 -month period January, 1931-September, 1939.

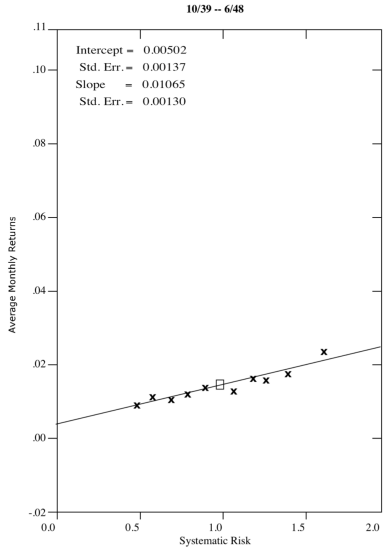


Figure 9 Average Excess monthly returns versus systematic risk for the 105 - month period October, 1939-June, 1948.

CAPM in practice is too flat

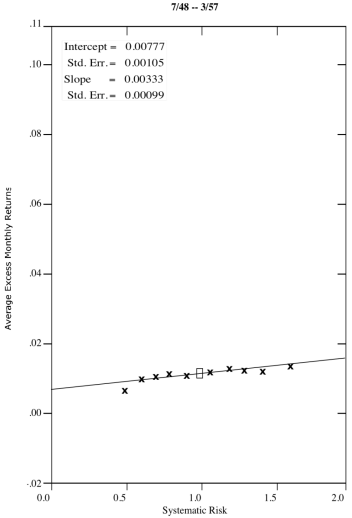


Figure 4 Average Excess monthly returns versus systematic risk for the 105 -month period July, 1948-March, 1957. Symbols as in Figure 1.

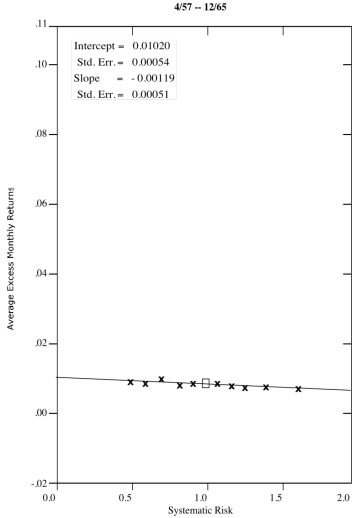
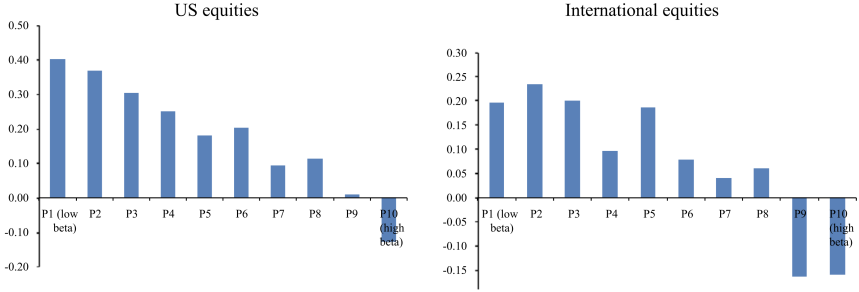


Figure 4 Average Excess monthly returns versus systematic risk for the 105 -month period April, 1957 - December, 1965. Symbols as in Figure 1.

CAPM in practice is too flat, even in 2014

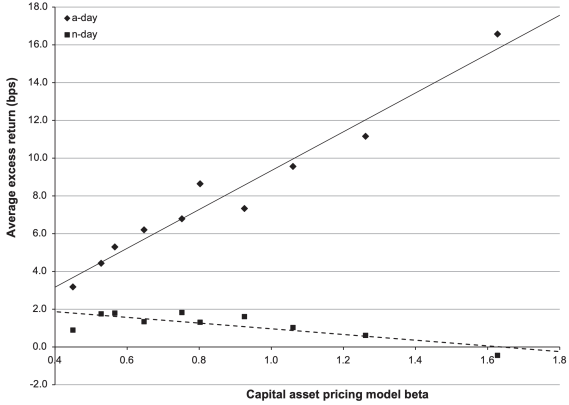
- Average monthly alpha, in % for 10 portfolios sorted on CAPM beta.



Source: Frazzini, Andrea and Pedersen, Lasse Heje, 2014, Betting against beta, *Journal of Financial Economics* 111, 1-25.

CAPM in practice is too flat, even in 2014, unless you look at the right dates.

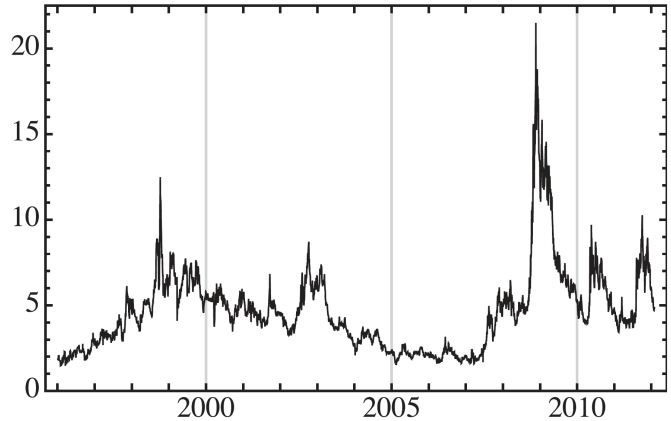
- CAPM fit on Federal Open Market Committee, inflation & unemployment data announcement days.



Source: Savor, Pavel and Wilson, Mungo, 2014, Asset pricing: A tale of two days, *Journal of Financial Economics* 113, 171-201.

But which market risk premium to use is also problematic.

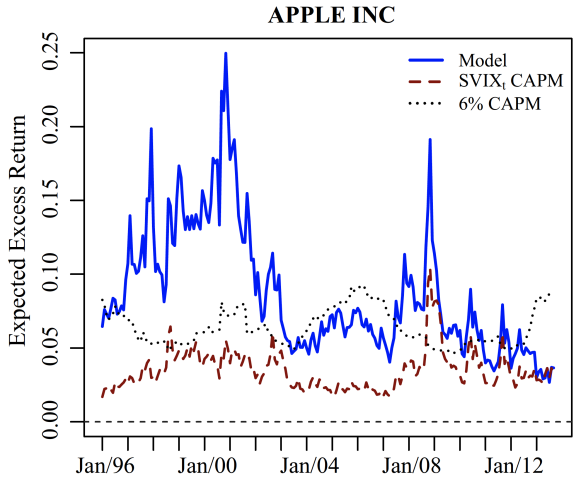
- Lower bound on the equity risk premium implied by option prices. It's not always 6%, sometimes it's 20%!



Source: Martin, Ian, 2017, What is the expected return on the market?, *The Quarterly Journal of Economics* 132(1), 367-433.

So what is a firm's cost of equity?

- Apple Inc.'s cost of equity, implied by option prices, in blue.



Source: Martin, Ian and Wagner, Christian, 2019, What is the expected return on a stock?, *The Journal of Finance* 74(4), 1887-1929.

Summary



To summarize, the CAPM is a mess.

However, all equity valuations still rely on the CAPM to estimate the firm's cost of equity.
The most common method to estimate beta is 5 years, monthly (60 observations). However, we have seen that this is very noisy.

Nevertheless, the CAPM is still useful as a framework to think about the relationship between risk and return, and helps discipline managers when thinking about the cost of capital.

For the risk-free rate, the standard choice is to use 3-3.5% and 5.5-6% for the risk premium.

Plan For Today

- Derivation (Supplementary)
- Interpretation / Formula's
- Appendix: CAPM in Practice
- Appendix: Re-levering β

Debt and the Return on Equity

	2025	2026	Return
Equity	100	110	+10%
Debt	0	0	–
Assets (A)	100	110	+10%

	2025	2026	Return
Equity	50	60	+20%
Debt	50	50	0%
Assets (A)	100	110	+10%



Debt makes equity riskier.

Implications on CAPM

Suppose I want to start a new restaurant business. I want to know the average cost of capital for operating in this sector, but the equity beta of many of the companies is affected by their capital structure.

We call the beta that we use in the CAPM as the levered beta, and the beta of the underlying business itself as the asset beta or the unlevered beta.

Unlevered Cost of Capital



The standard CAPM formula uses the levered beta.

$$\mathbb{E}[r_E] = r^f + \beta(\mathbb{E}[r^m] - r^f)$$

where r_E refers to the equity cost of capital, or the levered cost of equity.

The unlevered cost of capital or the return on assets is given by

$$E[r_U] = r^f + \beta_U(\mathbb{E}[r^m] - r^f)$$

where β_U is the unlevered beta.

Unlevered Beta

The formula for unlevered beta is

$$\beta_U = \frac{E}{E+D}\beta_E + \frac{D}{E+D}\beta_D$$

- E: the market value of equity
- D: the book/market value of debt
- β_E: standard CAPM beta
- β_D: debt beta (generally assume $\beta_D = 0$)

Levered Beta in the Airline Business

Firm	Debt ($M)	MC	D/E	$E/(E+D)$	Equity β	Unlevered β
Southwest	5,981	21,374	0.28	0.78	1.06	0.83
Delta	20,274	45,316	0.45	0.69	1.38	0.95
United	31,036	36,200	0.86	0.54	1.55	0.83

*For fiscal year 2025. MC = market cap ($M). Unlevered $\beta_U = \frac{E}{E+D}\beta_E + \frac{D}{E+D}\beta_D$ with $\beta_D = 0$. American Airlines is excluded due to the excessive debt it took to survive Covid-19, with a D/E ratio around 3.6.

Some textbooks/resources calculate unlevered beta using:
$\beta_U = \beta_E / \left[1 + (1-\tau)D/E\right]$ where τ is the tax-rate. Often, the $(1-\tau)$ term is omitted.

Re-levering beta

In your interviews for consulting / investment banking, the topic of re-levering beta often comes up.

In a hypothetical scenario, I would like to enter a new industry, so I need to know the industry's unlevered cost of capital (the cost of doing business in that industry). However, I need to then adjust for my own company's capital structure so I can properly compute the NPV of projects. Once you compute the unlevered beta, calculating the re-levered beta is just one step away,

$$\beta = \beta_U \times \Big[1 + (1 - \tau)D/E\Big]$$

where D/E is the target debt-to-equity ratio of the parent company. Then, plug that back into the CAPM to get the equity cost of capital.

Re-levering beta

Firm	Debt ($M)	MC	D/E	$E/(E+D)$	Equity β	Unlevered β
Southwest	5,981	21,374	0.28	0.78	1.06	
Delta	20,274	45,316	0.45	0.69	1.38	
United	31,036	36,200	0.86	0.54	1.55	

Suppose I plan to launch a low-cost carrier in the U.S., (like easyJet). Compute my new equity cost of capital assuming a target D/E ratio of 0.35, assume $\tau = 0$, $r^f = 3\%$ and the market risk premium = 5%.

Steps:

i Compute the unlevered beta for each company in the table of "comparables".

ii Take the average unlevered beta to get the asset beta for this industry

iii Apply the formula for re-levering beta, then plug into CAPM

$\beta_U = 0.87,\ \beta = 1.18,\ r_E = 0.089$